The nonlinear Schrodinger equation with a potential

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We consider the cubic nonlinear Schrodinger equation with a potential in one space dimension. Under the assumptions that the potential is generic, sufficiently localized, with no bound states, we obtain the long-time asymptotic behavior of small solutions. In particular, we prove that, as time goes to infinity, solutions exhibit nonlinear phase corrections that depend on the scattering matrix associated to the potential.

The proof of our result is based on the use of the distorted Fourier transform - the so-called Weyl-Kodaira-Titchmarsh theory - a precise understanding of the ``nonlinear spectral measure'' associated to the equation, and nonlinear stationary phase arguments and multilinear estimates in this distorted setting.